 Göteborg ITP preprint

11D supergravity at $\mathcal{O}(l^3)$

Dimitrios Tsimpis

Institute for Theoretical Physics
Göteborg University and
Chalmers University of Technology
SE-412 96 Göteborg, Sweden

ABSTRACT: We compute certain spinorial cohomology groups controlling possible supersymmetric deformations of eleven-dimensional supergravity up to order l^3 in the Planck length. At $\mathcal{O}(l)$ and $\mathcal{O}(l^2)$ the spinorial cohomology groups are trivial and therefore the theory cannot be deformed supersymmetrically. At $\mathcal{O}(l^3)$ the corresponding spinorial cohomology group is generated by a nontrivial element. On an eleven-dimensional manifold M such that $p_1(M) \neq 0$, this element corresponds to a supersymmetric deformation of the theory, which can only be redefined away at the cost of shifting the quantization condition of the four-form field strength.

KEYWORDS: .

Contents

1. Introduction and summary

In search of signatures of purely M-theoretic effects one may try to go beyond the limiting approximation of ordinary 11D supergravity, by including higher-order derivative (curvature) corrections. Despite considerable progress, however, a tractable microscopic definition of M-theory (see [1, 2] for reviews) on general backgrounds is still lacking and one would therefore have to resort to indirect computational methods. Supersymmetry, provided it will prove restrictive enough, is at present our best hope for addressing such corrections directly in eleven dimensions. The problem was analyzed within the framework of eleven-dimensional superspace in [3], producing some partial results. For a review of the literature on R^4 corrections in type II string theory and an attempt at lifting string-theory results to eleven dimensions, see [4, 5].

Addressing the issue of higher-order superinvariants can be done systematically using spinorial-cohomology methods. The concept of spinorial cohomology (SC) was introduced in [6] and further elaborated in [7]. A purely tensorial definition was subsequently given in [8]. SC has found a number of applications in ten-dimensional SYM [6, 9, 10] and eleven-dimensional supergravity [7, 8]. The $\mathcal{O}(l^4)$ corrections[1] to the worldvolume theory of the membrane in eleven dimensions were derived using SC in the context of the superembedding formalism, in [11]. Similar methods were used recently in investigating higher-order corrections to the world-volume theory of the D9 brane [12]. SC with unrestricted coefficients can be shown to be equivalent to pure-spinor [13, 14] cohomology, which was

[1] In this paper we use l to denote the Planck length.

recently used by Berkovits in the covariant quantization of the superstring [15, 16]. An alternative method of computing the cohomology by relaxing the pure-spinor constraint was considered in [17].

Although the first supersymmetric correction to eleven-dimensional supergravity is expected to occur at order l^6, the existence of superinvariants already at lower orders in the Planck length has not been rigorously excluded. In this note we examine the possibility of supersymmetric corrections to eleven-dimensional supergravity, up to order l^3. As explained in the following, such corrections are controlled by certain spinorial-cohomology groups, which we compute to order l^3 in section 4. We find that up to order l^2 the relevant groups are trivial and therefore there are no possible supersymmetric deformations of the theory.

At l^3 we find that the corresponding spinorial-cohomology group is one-dimensional, therefore there exists a unique superinvariant at this order. On a (spin, orientable) spacetime manifold M such that the first Pontryagin class $p_1(M)$ vanishes, this superinvariant can be removed by an appropriate field redefinition of the three-form superpotential (C). However, on a topologically nontrivial spacetime such that $p_1(M) \neq 0$, the superinvariant cannot be redefined away without changing the quantization condition of the four-form field strength ($G = dC$, locally). The latter is determined, by requiring quantum consistency of the theory, to be of the form [18]

$$[\frac{G}{2\pi l^3}] - \frac{1}{4}p_1(M) \in H^4(M, \mathbb{Z}) \ , \tag{1.1}$$

where the brackets denote the cohomology class. Recently a lot of effort has been invested in understanding the precise mathematical nature of the C-field on spacetime manifolds of nontrivial topology (see [18, 19, 20, 21, 22, 23] for an inexhaustive list of related literature). It would clearly be of interest to understand the significance of our result in relation to this issue.

In the next section we review the tensorial definition of SC. Relevant aspects of the superspace formulation of ordinary eleven-dimensional supergravity are included in section 3 and in the appendix. Section 4 contains the computation of the spinorial-cohomology groups to order l^3.

2. Spinorial cohomology

In this section we give a summary of the tensorial definition of spinorial cohomology for superforms (see [8] for a more detailed discussion). We will suppose that the tangent bundle is a direct sum of the odd and even bundles and that the supermanifold is equipped with a connection with Lorentzian structure group.

The space of forms admits a natural bigrading according to the degrees and Grassmann character of the forms. Let us denote by $\Omega^{p,q}$ the space of superforms ω with p even and q odd components

$$\omega_{a_1 \ldots a_p \alpha_1 \ldots \alpha_q} \in \Omega^{p,q} \ . \tag{2.1}$$

The exterior derivative d has the following action on $\Omega^{p,q}$,

$$d: \quad \Omega^{p,q} \rightarrow \Omega^{p+1,q} \oplus \Omega^{p,q+1} \oplus \Omega^{p-1,q+2} \oplus \Omega^{p+2,q-1} \ . \tag{2.2}$$

Following [24] we split d into its various components with respect to the bigrading

$$d = d_0 + d_1 + \tau_0 + \tau_1 \ , \tag{2.3}$$

where d_0 (d_1) is the even (odd) derivative with bidegrees $(1,0)$ and $(0,1)$ respectively, while τ_0 and τ_1 have bidegrees $(-1,2)$ and $(2,-1)$. These two latter operators are purely algebraic and involve the dimension-zero and dimension-three-halves components of the torsion tensor respectively. The fact that $d^2 = 0$ implies in particular that $\tau_0^2 = 0$. We can therefore consider the τ_0 cohomology groups

$$H_\tau^{p,q} = \{\omega \in \Omega^{p,q} | \ \tau_0 \omega = 0\} / \{\tau_0 \lambda, \ \lambda \in \Omega^{p+1,q-2}\} \ . \tag{2.4}$$

We can now define a spinorial derivative d_F which will act on elements of $H_\tau^{p,q}$. For $\omega \in [\omega] \in H_\tau^{p,q}$ we set

$$d_F[\omega] := [d_1 \omega] \ . \tag{2.5}$$

It is easy to check that this is well-defined, i.e. $d_1\omega$ is τ_0-closed, and $d_F[\omega]$ is independent of the choice of representative. Moreover it is straightforward to show that d_F is nilpotent. The spinorial cohomology groups are defined as

$$\begin{aligned} H_F^{p,q} &:= H^{p,q}(d_F|H_\tau) \\ &:= \{\omega \in H_\tau^{p,q} | \ d_F \omega = 0\} / \{d_F \lambda, \ \lambda \in H_\tau^{p+1,q-2}\} \ . \end{aligned} \tag{2.6}$$

If we are interested in deformations of the theory, we need to consider the above cohomology groups with coefficients restricted to be tensorial functions of the physical fields of the theory and their derivatives. We will denote these groups by $H_F^{p,q}(phys)$.

3. Undeformed 11D supergravity

Eleven-dimensional supergravity [25] admits a superspace formulation [26, 27]. Let $A = (a,\alpha)$; $a = 0 \dots 10, \ \alpha = 1 \dots 32$, be a flat superspace index and let $E^A = (E^a, E^\alpha)$ be the coframes of the $(11|32)$ supermanifold. Moreover, let us introduce a connection one-form $\Omega_A{}^B$ with Lorentzian structure group. The supertorsion and supercurvature are given by

$$\begin{aligned} T^A &= DE^A := dE^A + E^B \Omega_B{}^A = \frac{1}{2} E^C E^B T_{BC}{}^A \\ R_A{}^B &= d\Omega_A{}^B + \Omega_A{}^C \Omega_C{}^B = \frac{1}{2} E^D E^C R_{CD,A}{}^B \end{aligned} \tag{3.1}$$

and obey the Bianchi identities

$$\begin{aligned} DT^A &= E^B R_B{}^A \\ DR_B{}^A &= 0 \ . \end{aligned} \tag{3.2}$$

Note that for a Lorentzian structure group the second Bianchi identity follows from the first [28]. In a purely geometrical definition in terms of the supertorsion, it was shown in [29] that the equations of motion of 11D supergravity follow from the constraint

$$T_{\alpha\beta}{}^{a} = -i(\gamma^{a})_{\alpha\beta} \ . \tag{3.3}$$

In this formulation the physical fields of the theory, the graviton, the gravitino and the three-form potential, appear through their covariant field strengths. Namely, the curvature $R_{abc}{}^{d}$ is identified with the top component of the supercurvature, the gravitino field-strength $T_{ab}{}^{\alpha}$ is identified with the dimension three-halves component of the supertorsion, while the four-form field strength G_{abcd} appears in the dimension-one components of the supertorsion and supercurvature. For completeness we have included in the appendix all nonzero components of the supertorsion and the supercurvature, the action of the spinorial derivative on the physical field strengths and their equations of motion.

The theory admits an alternative formulation in terms of a closed superfour-form G_{ABCD},

$$D_{[A}G_{BCDE\}} + 2T_{[AB|}{}^{F}G_{F|CDE\}} = 0 \ . \tag{3.4}$$

In this description, undeformed supergravity is recovered by imposing the constraint that the lowest component of the superfour-form vanishes [3, 30],

$$G_{\alpha\beta\gamma\delta} = 0 \ . \tag{3.5}$$

4. Deformations

It was pointed out in [7, 8] that from the point of view of the superfour-form formulation of supergravity, the physical fields of the theory are elements of $H_F^{0,3}$ while supersymmetric deformations are parameterized by elements $G_{\alpha\beta\gamma\delta}$ such that

$$G_{\alpha\beta\gamma\delta} \in H_F^{0,4}(phys) \ . \tag{4.1}$$

The content of equation (4.1) can be restated explicitly as follows [8]: Supersymmetric deformations of the theory are parameterized by objects $G_{\alpha\beta\gamma\delta}$ such that

$$\begin{aligned} G_{\alpha\beta\gamma\delta} &= \frac{1}{8}(\gamma^{a_1a_2})_{(\alpha\beta}(\gamma^{b_1b_2})_{\gamma\delta)}A_{a_1a_2;b_1b_2} \\ &+ \frac{1}{240}(\gamma^{a_1\ldots a_5})_{(\alpha\beta}(\gamma^{b_1b_2})_{\gamma\delta)}B_{a_1\ldots a_5;b_1b_2} \\ &+ \frac{1}{28800}(\gamma^{a_1\ldots a_5})_{(\alpha\beta}(\gamma^{b_1\ldots b_5})_{\gamma\delta)}C_{a_1\ldots a_5;b_1\ldots b_5} \quad , \end{aligned} \tag{4.2}$$

where $A,\ B,\ C$ are irreducible (p,q)-tensors satisfying

$$\begin{aligned} I_A &:= D_\alpha\left(A - \frac{7}{5}B\right)|_{(02001)} = 0 \\ I_B &:= D_\alpha\left(B - \frac{3}{10}C\right)|_{(01003)} = 0 \\ I_C &:= D_\alpha C|_{(00005)} = 0 \ . \end{aligned} \tag{4.3}$$

The vertical bars in (4.3) denote projection onto the representations indicated by the corresponding Dynkin weights [2]. Moreover, $G_{\alpha\beta\gamma\delta}$ is defined modulo shifts of the form

$$G_{\alpha\beta\gamma\delta} \rightarrow G_{\alpha\beta\gamma\delta} + D_{(\alpha}C_{\beta\gamma\delta)} \ , \tag{4.4}$$

where

$$C_{\alpha\beta\gamma} = (\gamma^{ab})_{(\alpha\beta|}V_{ab|\gamma)} + (\gamma^{a_1\dots a_5})_{(\alpha\beta|}U_{a_1\dots a_5|\gamma)} \tag{4.5}$$

and $V_{ab\alpha}$, $U_{a_1\dots a_5\alpha}$ are irreducible (gamma-traceless) tensor-spinors. All fields in (4.2), (4.5) are to be understood as tensorial functions of the physical field-strengths of the theory and their derivatives.

Note that once a $G_{\alpha\beta\gamma\delta} \in H_F^{0,4}(phys)$ has been determined, this information can be fed into the Bianchi identities in order to derive the equations of motion, and therefore the Lagrangian, of the deformed theory.

4.1 Deformations at $\mathcal{O}(l)$, $\mathcal{O}(l^2)$ and $\mathcal{O}(l^3)$

The canonical dimensions of the physical field-strengths of the theory are as follows

$$\begin{aligned} [G_{abcd}] &= l^{-1} \\ [T_{ab}{}^{\alpha}] &= l^{-3/2} \\ [R_{abcd}] &= l^{-2} \ . \end{aligned} \tag{4.6}$$

The dimensions of the fields in (4.2), (4.5) are

$$\begin{aligned} [A] = [B] = [C] &= l \\ [V] = [U] &= l^{3/2} \ . \end{aligned} \tag{4.7}$$

At any given order $\mathcal{O}(l^n)$, $n \geq 0$, these fields should be expressed as functions $l^n f(G, T, R)$ of the physical field-strengths, where f does not depend on l.

Clearly, at order $\mathcal{O}(l)$ there can be no fields A, B, C satisfying these requirements. At order $\mathcal{O}(l^2)$ the combination $l^2 G_{abcd}$ has the same dimension as A, B, C, but transforms in the 'wrong' representation. We therefore conclude that at $\mathcal{O}(l)$, $\mathcal{O}(l^2)$ the group $H_F^{0,4}(phys)$ is trivial and the theory does not admit supersymmetric deformations.

At order $\mathcal{O}(l^3)$ dimensional analysis and representation theory reveals that the most general expression for the fields A, B, C is

$$\begin{aligned} A_{a_1a_2,b_1b_2} &= l^3 \left(c_1 \ G_{a_1a_2}{}^{ij}G_{b_1b_2ij} + c_2 \ R_{a_1a_2b_1b_2}\right)|_{(02000)} \\ B_{a_1\dots a_5,b_1b_2} &= l^3 \left(c_3 \ \varepsilon_{a_1\dots a_5}{}^{ijklmn}G_{ijkl}G_{b_1b_2mn}\right)|_{(01002)} \\ C_{a_1\dots a_5,b_1\dots b_5} &= 0 \ . \end{aligned} \tag{4.8}$$

[2] Explicit expressions for the projections and an explanation of the representation-theoretical notation used here can be found in [8], to which the reader is referred for more details.

Similarly for the fields U, V we get

$$\begin{aligned} V_{ab\alpha} &= l^3(c_4\ T_{ab\alpha}) \\ U_{a_1\ldots a_5\alpha} &= 0\ , \end{aligned} \tag{4.9}$$

where $c_1 \ldots c_4$ are arbitrary real constants at this stage. For the constraints in (4.3) we have

$$\begin{aligned} I_A &= l^3 \left(c_5\ G_{a_1a_2}{}^{ij}(\gamma_{b_1b_2}T_{ij})_\alpha\right)|_{(02001)} \\ I_B &= 0 \\ I_C &= 0\ , \end{aligned} \tag{4.10}$$

where c_5 is a linear combination of c_1, c_2, c_3 which can be determined from (4.3), (4.8). The action of the spinorial derivative on the physical field-strengths is given in the appendix. We see that the I_B, I_C constraints are automatically satisfied. Moreover, imposing $I_A = 0$ fixes the ratio c_1/c_3 in terms of c_2/c_3. Finally, using the freedom (4.4) it is straightforward to see from (4.5), (4.9) that the term in $A_{a_1a_2,b_1b_2}$ proportional to $R_{a_1a_2b_1b_2}|$ (cf. (4.8)) can be redefined away for a suitable c_4. The above discussion is illustrated in figure 1.

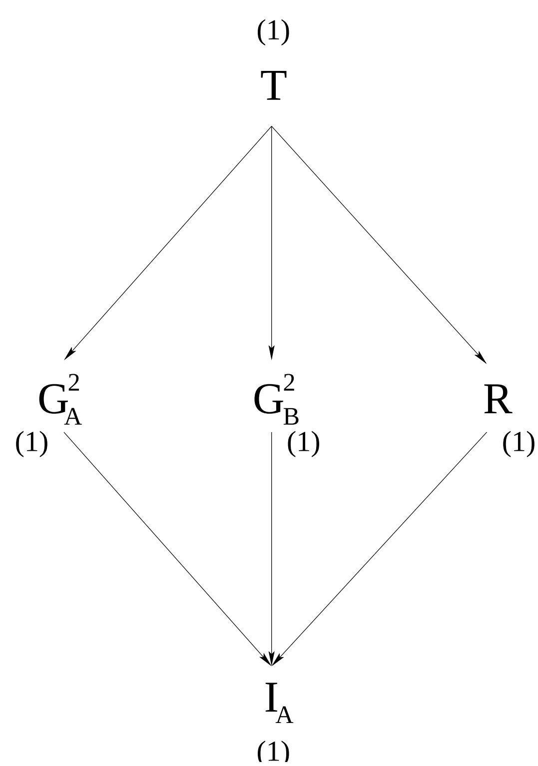

Figure 1: Spinorial cohomology at l^3. The first, second and third rows depict schematically all possible terms in V, $A \oplus B$ and I_A respectively. The arrows indicate the action of d_F. Multiplicities are denoted by the numbers in parentheses.

To summarize, we have found that $H_F^{0,4}(phys)$ is generated by a certain linear combination of the two G^2 terms in (4.8). This result can be understood alternatively as follows: at order $\mathcal{O}(l^3)$ one can consider shifts of the superfour-form G of the type

$$G \to G_\beta := G - \frac{\beta l^3}{4\pi} trR^2 \ , \tag{4.11}$$

where

$$trR^2 := \frac{1}{4} E^D E^C E^B E^A R_{AB}{}^{ab} R_{CDba} \tag{4.12}$$

and β is an arbitrary real parameter. Note that the top component of trR^2 is proportional (taken at $\theta = 0$) to the first Pontryagin class $p_1(M)$ of the eleven-dimensional spacetime manifold M,

$$p_1(M) = -\frac{1}{8\pi^2} trR^2 \ . \tag{4.13}$$

The lowest, purely spinorial, component of trR^2 is equal to a certain linear combination of G^2 terms, as can be seen from (A.2) of the appendix. It could be brought to the form (4.2) by a field redefinition, but it will be convenient not to do so here. If G is closed so is G_β, by virtue of the second Bianchi identity in (3.2). Therefore, the resulting theory can be obtained from the undeformed one simply by replacing G with G_β and this is the *only* resulting modification to the equations of motion. In other words, the generating element of $H_F^{0,4}(phys)$ can be absorbed by a transformation of the form (4.11).

Locally (4.11) can be expressed as a shift in the superthree-form potential

$$C \to C_\beta := C - \frac{\beta l^3}{4\pi} Q \ , \tag{4.14}$$

where Q, $dQ = trR^2$, is the Chern-Simons form

$$Q := tr(\Omega d\Omega + \frac{2}{3}\Omega^3) \ . \tag{4.15}$$

Note however, that this is admissible as a field redefinition only in the case $p_1(M) = 0$. In the generic case, $p_1(M) \neq 0$, the cohomology classes of G, G_β are different, and the two theories related by the shift (4.14) are inequivalent.

At the level of classical actions, there is a one-parameter family of supersymmetric theories given by [3]

$$S_\beta = \frac{1}{l^9} \int \Big(R^*1 - \frac{1}{2} G_\beta \wedge *G_\beta + \frac{1}{6} C_\beta \wedge G_\beta \wedge G_\beta + \mathcal{O}(l^6) \Big)_{\theta=0} \ . \tag{4.16}$$

Note that unless one postulates an unconventional parity transformation law for the four-form G, (4.16) is not parity-invariant. The actions (4.16) are related to the ordinary supergravity action S_0 by a shift in the three-form potential that changes the cohomology class of G:

$$[\frac{G_\beta}{2\pi l^3}] = [\frac{G}{2\pi l^3}] + \beta p_1(M) \ . \tag{4.17}$$

[3]For simplicity we concentrate on the bosonic part; we use the same letters (C, G, R) for the top components of the corresponding superforms.

On the other hand, quantum mechanical consistency of the theory forces G to obey the quantization condition [18]

$$[\frac{G}{2\pi l^3}] - \frac{1}{4}p_1(M) \in H^4(M,\mathbb{Z}) \tag{4.18}$$

and therefore G_β obeys a shifted quantization condition dictated by (4.17),(4.18).

Acknowledgments

I am indebted to P. Howe for valuable discussions. This work is supported by EU contract HPRN-CT-2000-00122.

A. Undeformed 11D supergravity in superspace

The nonzero components of the supertorsion and supercurvature of undeformed 11D supergravity are given by

$$\begin{aligned} T_{\alpha\beta}{}^{c} &= -i(\gamma^c)_{\alpha\beta} \\ T_{a\beta}{}^{\gamma} &= -\frac{1}{36}\left((\gamma^{bcd})_\beta{}^\gamma G_{abcd} + \frac{1}{8}(\gamma_{abcde})_\beta{}^\gamma G^{abcd}\right) \end{aligned} \tag{A.1}$$

and

$$\begin{aligned} R_{\alpha\beta ab} &= \frac{i}{6}\left((\gamma^{cd})_{\alpha\beta}G_{abcd} + \frac{1}{24}(\gamma_{abcdef})_{\alpha\beta}G^{cdef}\right) \\ R_{\alpha bcd} &= \frac{i}{2}\left((\gamma_b T_{cd})_\alpha + (\gamma_c T_{bd})_\alpha - (\gamma_d T_{bc})_\alpha\right) \ . \end{aligned} \tag{A.2}$$

Note that the Lorentz condition implies

$$R_{AB\alpha}{}^{\beta} = \frac{1}{4}R_{ABcd}(\gamma^{cd})_\alpha{}^\beta \ . \tag{A.3}$$

The action of the spinorial derivative on the physical field strengths and their equations of motion are given by

$$\begin{aligned} D_\alpha G_{abcd} &= 6i(\gamma_{[ab}T_{cd]})_\alpha \\ D_\alpha T_{ab}{}^{\beta} &= \frac{1}{4}R_{abcd}(\gamma^{cd})_\alpha{}^\beta - 2D_{[a}T_{b]\alpha}{}^\beta - 2T_{[a|\alpha}{}^{\epsilon}T_{|b]\epsilon}{}^\beta \\ D_\alpha R_{abcd} &= 2D_{[a|}R_{\alpha|b]cd} - T_{ab}{}^{\epsilon}R_{\epsilon\alpha cd} + 2T_{[a|\alpha}{}^{\epsilon}R_{\epsilon|b]cd} \end{aligned} \tag{A.4}$$

and

$$\begin{aligned} D_{[a}G_{bcde]} &= 0 \\ D^f G_{fabc} &= -\frac{1}{2(4!)^2}\varepsilon_{abcd_1\ldots d_8}G^{d_1\ldots d_4}G^{d_5\ldots d_8} \\ (\gamma^a T_{ab})_\alpha &= 0 \\ R_{ab} - \frac{1}{2}\eta_{ab}R &= -\frac{1}{12}\left(G_{adfg}G_b{}^{dfg} - \frac{1}{8}\eta_{ab}G_{dfge}G^{dfge}\right) \ . \end{aligned} \tag{A.5}$$

The above equations can be integrated to an action whose bosonic part reads

$$S = \frac{1}{l^9} \int \left(R^* 1 - \frac{1}{2} G \wedge * G + \frac{1}{6} C \wedge G \wedge G \right)_{\theta=0} . \tag{A.6}$$

References

[1] M. J. Duff, "The World In Eleven Dimensions: Supergravity, Supermembranes And M-Theory," Bristol, UK: IOP (1999).

[2] W. Taylor, "M(atrix) theory: Matrix quantum mechanics as a fundamental theory," Rev. Mod. Phys. **73** (2001) 419; hep-th/0101126.

[3] M. Cederwall, U. Gran, M. Nielsen and B. E. Nilsson, "Manifestly supersymmetric M-theory," hep-th/0007035, JHEP **0010** (2000) 041; "Generalised 11-dimensional supergravity," hep-th/0010042.

[4] K. Peeters, P. Vanhove and A. Westerberg, "Supersymmetric higher-derivative actions in ten and eleven dimensions, the associated superalgebras and their formulation in superspace," hep-th/0010167, Class. Quant. Grav. **18** (2001) 843.

[5] K. Peeters, P. Vanhove and A. Westerberg, "Chiral splitting and world-sheet gravitinos in higher-derivative string amplitudes," Class. Quant. Grav. **19** (2002) 2699; hep-th/0112157.

[6] M. Cederwall, B. E. Nilsson and D. Tsimpis, "The structure of maximally supersymmetric Yang-Mills theory: Constraining higher-order corrections," JHEP **0106** (2001) 034; hep-th/0102009.

[7] M. Cederwall, B. E. Nilsson and D. Tsimpis, "Spinorial cohomology and maximally supersymmetric theories," JHEP **0202** (2002) 009; hep-th/0110069.

[8] P. S. Howe and D. Tsimpis, "On higher-order corrections in M theory," JHEP **0309** (2003) 038; hep-th/0305129.

[9] M. Cederwall, B. E. Nilsson and D. Tsimpis, "D = 10 super-Yang-Mills at $O(\alpha'^2)$," JHEP **0107** (2001) 042; hep-th/0104236.

[10] M. Cederwall, B. E. Nilsson and D. Tsimpis, "Spinorial cohomology of abelian d = 10 super-Yang-Mills at $O(\alpha'^3)$," JHEP **0211** (2002) 023; hep-th/0205165.

[11] P. S. Howe, S. F. Kerstan, U. Lindstrom and D. Tsimpis, "The deformed M2-brane," JHEP **0309** (2003) 013; hep-th/0307072.

[12] J. M. Drummond and S. F. Kerstan, "Kappa-symmetric derivative corrections to D-brane dynamics," hep-th/0407145.

[13] P. S. Howe, "Pure Spinors Lines In Superspace And Ten-Dimensional Supersymmetric Theories," Phys. Lett. B **258** (1991) 141 [Addendum-ibid. B **259** (1991) 511].

[14] P. S. Howe, "Pure spinors, function superspaces and supergravity theories in ten-dimensions and eleven-dimensions," Phys. Lett. B **273** (1991) 90.

[15] N. Berkovits, "Super-Poincare covariant quantization of the superstring," JHEP **0004** (2000) 018; hep-th/0001035.

[16] N. Berkovits, "Covariant quantization of the superparticle using pure spinors," JHEP **0109** (2001) 016; hep-th/0105050.

[17] P. A. Grassi, G. Policastro, M. Porrati and P. Van Nieuwenhuizen, "Covariant quantization of superstrings without pure spinor constraints," JHEP **0210** (2002) 054; hep-th/0112162. P. A. Grassi, G. Policastro and P. van Nieuwenhuizen, "The massless spectrum of covariant superstrings," JHEP **0211** (2002) 001; hep-th/0202123.

[18] E. Witten, "On flux quantization in M-theory and the effective action," J. Geom. Phys. **22** (1997) 1; hep-th/9609122.

[19] D. E. Diaconescu, G. W. Moore and E. Witten, "E(8) gauge theory, and a derivation of K-theory from M-theory," Adv. Theor. Math. Phys. **6** (2003) 1031; hep-th/0005090.

[20] D. E. Diaconescu, G. W. Moore and E. Witten, "A derivation of K-theory from M-theory," hep-th/0005091.

[21] G. W. Moore and E. Witten, "Self-duality, Ramond-Ramond fields, and K-theory," JHEP **0005** (2000) 032; hep-th/9912279.

[22] G. W. Moore, "Some comments on branes, G-flux, and K-theory," Int. J. Mod. Phys. A **16** (2001) 936; hep-th/0012007.

[23] E. Diaconescu, G. Moore and D. S. Freed, "The M-theory 3-form and E(8) gauge theory," hep-th/0312069.

[24] L. Bonora, K. Lechner, M. Bregola, P. Pasti and M. Tonin, "A Discussion Of The Constraints In N=1 Sugra-Sym In 10-D," Int. J. Mod. Phys. A **5** (1990) 461.

[25] E. Cremmer, B. Julia and J. Scherk, "Supergravity Theory In 11 Dimensions," Phys. Lett. B **76** (1978) 409.

[26] E. Cremmer and S. Ferrara, "Formulation Of Eleven-Dimensional Supergravity In Superspace," Phys. Lett. B **91** (1980) 61.

[27] L. Brink and P. S. Howe, "Eleven-Dimensional Supergravity On The Mass - Shell In Superspace," Phys. Lett. B **91** (1980) 384.

[28] N. Dragon, "Torsion And Curvature In Extended Supergravity," Z. Phys. C **2** (1979) 29.

[29] P. S. Howe, "Weyl superspace," hep-th/9707184, Phys. Lett. B **415** (1997) 149.

[30] H. Nishino and S. Rajpoot, "A note on embedding of M-theory corrections into eleven-dimensional superspace," Phys. Rev. D **64** (2001) 124016; hep-th/0103224.